Opgave C NAW 5/4 nr. 3 sep 2003

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The problem

Introduction.

See NAW 5/4/ nr. 3 sep 2003, opgave C

Solution.

Let the points A, B, C, D be defined by coordinates (x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4). As easily can be shown, we have $Z_1 = Z_2$ with $x_{Z_1} = x_{Z_2} = \frac{1}{4}\sum_{i=1}^{4} x_i$ and $y_{Z_1} = y_{Z_2} = \frac{1}{4}\sum_{i=1}^{4} y_i$.
Let S_1 be the centre of gravity of ABD and S_2 that of BCD, than we have $x_{S_1} = \frac{x_1+x_2+x_4}{3}$, $y_{S_1} = \frac{y_1+y_2+y_4}{3}$, $x_{S_2} = \frac{x_2+x_3+x_4}{3}$ and $y_{S_2} = \frac{y_2+y_3+y_4}{3}$.
We define A_1 as the 'area' of ABD, A_2 the 'area' of BCD and the weighting factors $p_1 = \frac{A_1}{A_1+A_2}$ and $p_2 = \frac{A_2}{A_1+A_2}$. According to a more or less well known result we can calculate A_1 and A_2 from the coordinates:
$A_1 = \frac{1}{2}((y2 - y1)(x_1 + x_2) + (y_4 - y_2)(x_4 + x_2) + (y_1 - y_4)(x_1 + x_4))$ and
$A_2 = \frac{1}{2}((y3 - y2)(x_3 + x_2) + (y_4 - y_3)(x_4 + x_3) + (y_2 - y_4)(x_2 + x_4))$.
Now we can calculate Z_3 with $x_{Z_3} = p_1 \cdot x_{S_1} + p_2 \cdot x_{S_2}$ and $y_{Z_3} = p_1 \cdot y_{S_1} + p_2 \cdot y_{S_2}$.
If $Z_1 = Z_2 = Z_3$ we have equations

$$(4p_1 - 3)x_1 + x_2 + (1 - 4p_1)x_3 + x_4 = 0 \tag{1}$$

and

$$(4p_1 - 3)y_1 + y_2 + (1 - 4p_1)y_3 + y_4 = 0 \tag{2}$$

Without loss of generality we may state that $A(-a, 0), B(0, b), C(x_3, y_3)$ and $D(0, d)$ with $a > 0$ and $d > b$. We have $A_1 = \frac{1}{2}a(d - b)$, $A_2 = \frac{1}{2}x_3(d - b)$ and $p_1 = \frac{a}{a+x_3}$.
When we solve the above equation (1) for x_3 we find $x_3 = \pm a$. The only solution that holds is $x_3 = a$. With the second equation we find $y_3 = b + d$.
So ABCD is a parallellogram